International Journal of Heat and Mass Transfer 52 (2009) 4090–4094

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Influence of boundary conditions on the structure of laminar boundary layer with hydrogen combustion on a permeable surface

E.P. Volchkov, V.V. Terekhov *, V.I. Terekhov

Kutateladze Institute of Thermophysics SB RAS, 630090, Novosibirsk, Russia

A R T I C L E I N F O

Article history:
Received 11 September 2008
Received in revised form 29 September 2008
Accepted 29 September 2008
Available online 4 May 2009

Keywords:
Laminar boundary layer
Porous blowing
Combustion
Heat and mass transfer
Boundary conditions

A B S T R A C T

A numerical study was performed to examine how thermal and diffusion boundary conditions affect the structure of laminar diffusion flame in air flow with porous blowing and combustion of hydrogen. Boundary conditions of two types were compared, with lengthwise-constant porous-wall temperature ($T_W = const$ throughout the whole range of blowing ratios), and with lengthwise-constant temperature of the fuel supplied into the main flow ($T' = const$). With a liquid fuel having constant evaporating-surface temperature, we deal with boundary conditions of the first type. With a gas fuel, as a rule, we encounter the regime with $T' = const$. It is shown that, in spite of the significant difference in velocity and temperature profiles, in both cases the surface friction coefficients have close values. Also, it is found that the wall heat flux exhibits a maximum if considered as a function of fuel supply intensity. Nonetheless, the function of relative heat transfer monotonically decreases with blowing intensity, much like it does in non-reactive flow.

1. Introduction

Combustion in boundary layer with porous fuel supply is governed by many factors and conditions that affect the flame structure, friction, and mass transfer. Most important parameters here are boundary conditions for heat and mass transfer on the porous surface. It is these parameters that define the flame-front position, the heat and mass fluxes, the flow pattern in the boundary layer, and the wall shear. Among the great variety of possible thermal and diffusion boundary conditions, two cases are most frequently encountered in practice, with lengthwise-constant wall temperature ($T_W = const$), and with lengthwise-constant temperature of the combustible gas injected into the main flow ($T' = const$ throughout the whole range of blowing ratios, see Fig. 1). The first situation is encountered in the combustion of a liquid fuel film whose surface temperature equals the saturation temperature. The problem of interest was given a detailed experimental study in [1], where the combustion of evaporating liquid ethanol was treated. Besides, the problem in this formulation was solved in [2] for laminar and turbulent flow modes with hydrogen blowing and burning. In [2], the wall temperature was assumed constant throughout the whole range of injected hydrogen concentrations. This somewhat hypothetical a case with hydrogen combustion at $T_W = const$, although hard to implement in practice, may prove useful in performing comparative analyses.

As a rule, a gas fuel is injected into the boundary layer at lengthwise-constant injected-gas temperature ($T' = const$). The constant T' condition is the same as the constant 'transferred substance state' from the well-known works of Spalding [3,4]. Here, the temperature of the streamlined surface varies in the downstream direction depending on the longitudinal distribution of the transverse flow of substance, defined by the injection intensity. At lengthwise-constant mass flux of the transverse flow, the wall concentration of injected substance increases to finally reach unity (regime with blown-off boundary layer). For a lengthwise-constant wall concentration of the fuel to be ensured, in laminar flow mode the blowing intensity has to decrease over the plate length as $\bar{j}_W \sim \mathrm{Re}_x^{-0.5}$. The study of such fuel supply regimes is also of interest for flame and heat- and mass-transfer control in flows with combustion.

The majority of experimental studies of boundary layers with fuel injection and combustion [5–8] were carried out on the assumption that the temperature and velocity of the injected gas flow were constant quantities. To date, no comparative analysis with the regime $T_W = const$ has been reported. Such an analysis would be of interest for comprehending basic regularities of heat- and mass-transfer processes proceeding under various thermal and concentration conditions.

The injection mode with lengthwise-constant injected-gas temperature has many specific features in comparison with the case of constant wall temperature. It was shown in [9] that at uniform gas blowing with $T' = const$, and also at blowing of a foreign gas into the main flow, the wall heat flux varies non-monotonically with

---

* Corresponding author. Fax: +7 383 3308480.
E-mail address: vt@itp.nsc.ru (V.V. Terekhov).

0017-9310/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2008.09.037

Nomenclature

b	blowing ratio, $b = 2j_W/\rho_0 u_0 c_{f0}$
c_f	friction coefficient, $c_f/2(\mu\partial u/\partial y)w/\rho_0 u_0^2$
j	mass flux of substance
H	mixture enthalpy
M	molecular weight of mixture
k	mass concentration
q	heat flux
Q	heat release due to chemical reaction
Pr	Prandtl number
Re	Reynolds number, $\mathrm{Re}_x = \rho_0 u_0 x/\mu_0$
Sc	Schmidt number
St	Stanton number, $\mathrm{St} = q_W/\rho_0 u_0 c_{p0}(T_0 - T')$
T	temperature
x	longitudinal coordinate
y	cross-stream coordinate
u, v	velocity-vector components
λ	heat conductivity of gas mixture
μ	mixture viscosity
ρ	mixture density
$\dot{\omega}_i$	rate of consumption/formation of the *i*-component
Subscripts	
fr	value in the flame front
w	value on the wall
0	parameter in the flow core
Superscript	
‘	parameter in the injected gas flow

blowing intensity, displaying a maximum at some blowing ratio. Such a behavior, exhibited by the heat fluxes, is observed both in laminar and turbulent flow modes. It would also be of interest to perform a similar analysis for boundary layer with combustion.

Thus, the purpose of the present study was to numerically investigate into the boundary-layer combustion process proceeding during porous blowing of hydrogen into laminar air flow. Primary attention was given to a comparative analysis of dynamic and thermal characteristics of the boundary layer realized at boundary conditions $T_W = const$ and $T' = const$ in the presence and absence of combustion.

2. Problem statement

In the present study, we examined heat and mass transfer of a porous flat plate exposed to a laminar stream of air with hydrogen combustion, the hydrogen being supplied into the main stream through the porous plate (Fig. 1). Particular attention was focused on different methods and modes of fuel supply. For instance, in treating the thermal problem, the case with lengthwise-constant wall temperature T_W and the case with lengthwise-constant injected-gas temperature T', in which the wall temperature is to be determined from heat balance equations, were addressed. It should be noted that both temperatures, T_W and T', were assumed to be constant throughout the whole range of blowing ratios. The mass-transfer conditions implied either lengthwise-constant fuel concentration k_W or lengthwise-constant mass rate of the injected fuel flow j_W. In the latter case, the wall concentration of the fuel is to be determined from substance balance equations. The flow velocity in the free stream was assumed to be $U_0 = 20$ m/s, and the temperature, $T_0 = 300$ K. In the case of fixed wall temperature, for the temperature T_W a value $T_W = 400$ K was adopted, and the same value, $T' = 400$ K, was assumed for the injected hydrogen flow. Several values of the relative mass rate of the injected flow were considered, covering the range $\bar{j}_W \sim 10^{-4}$–10^{-3}, as well as several values of lengthwise-constant wall fuel concentration, covering the range from 1% to 80%. The maximum Reynolds number over the plate length Re_x never exceeded 10^5, and the flow therefore was always laminar over its entire length. Besides, to study the effect due to the chemical process on heat and mass transfer, similar computations for the flow without combustion were carried out.

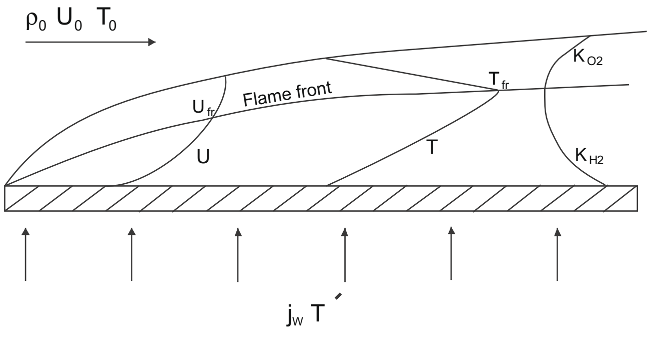

Fig. 1. Diagram of the reacting boundary-layer flow.

3. Governing equations and solution methods for the equations of boundary-layer flow with combustion

The general form of the conservation laws that govern the dynamics of multi-component gas mixtures with chemical reactions is well known [14]. However, combined solution of Navier–Stokes and chemical kinetics equations in their full statement is rather difficult a problem. Therefore, very often some simplifications, both for gas dynamics and heat and mass transfer and for the description of chemical processes, are used. In the present work, the problem under study was solved in the boundary layer approximation. The equations of two-dimensional stationary laminar boundary layer in a flow of mixed multi-component incompressible gases can be written as

$$\begin{aligned} &\frac{\partial \rho U}{\partial x} + \frac{\partial \rho V}{\partial y} = 0 \\ &\rho U \frac{\partial \Phi}{\partial x} + \rho V \frac{\partial \Phi}{\partial y} = \frac{\partial}{\partial y}\left(\Gamma \frac{\partial \Phi}{\partial y}\right) + S_\Phi \\ &\frac{\partial p}{\partial y} = 0 \end{aligned} \tag{1}$$

The values of the coefficients that enter these equations are given in Table 1.

It should be noted that, here, a simplified representation of diffusion fluxes for mixture components based on the Fick law is used. The diffusion and heat conduction coefficients, and also the viscosity and specific heats of individual substances were calculated from their polynomial representations [10]. The transfer

Table 1
Values of coefficients that enter the generalized transfer equation in the boundary layer approximation.

Quantity, Φ	Exchange coefficient, Γ	Source term, S_Φ
Flow velocity, U	μ	$-dp/dx$
Enthalpy, H	μ/Pr	Q
Concentration of the *i*th component, k_i	μ/Sc_i	$\dot{\omega}_i$

coefficients for the gas mixture were also calculated using the methods described in [10]. The chemical kinetics of hydrogen combustion with air was calculated according to the model of [11]. This kinetics was assumed to involve one global reaction, $2H_2 + O_2 \rightarrow 2H_2O$.

Even with such substantial simplifications, the governing equations for the problem admit an analytical solution only in rare simple cases. Therefore, main results as to boundary-layer combustion were obtained in the present study using numerical solution methods for non-linear mathematical-physics problems. In the present work, we used an approach described in detail in [12]. This approach is based on the transition from variables (x,y), in which the region occupied by the boundary layer had rather complex a shape, to variables (x,ω), where $\omega = \frac{\psi-\psi_W}{\psi_\infty-\psi_W}$ is the normalized stream function, and $\psi = \int_0^y \rho u\,dy$. In the latter coordinates, the integration domain becomes a rectangle $[0,L] \times [0,1]$. In addition, the use of such coordinates results in concentration of coordinate lines toward the wall in the (x,y)-plane, this condensation improving the accuracy in determining physical quantities in high-gradient flows. Another important benefit here is that, with the stream function used as an independent variable, the continuity equation turns out to be automatically satisfied. As a result, the generalized transfer equation in the coordinates (x,ω) assumes the form

$$\frac{\partial\Phi}{\partial x} + (a + b\omega)\frac{\partial\Phi}{\partial\omega} = \frac{\partial}{\partial\omega}\left(c\frac{\partial\Phi}{\partial\omega}\right) + \frac{S_\Phi}{\rho u}. \tag{2}$$

Here, a, b and c are coefficients that depend on the current values of the stream function at the domain boundaries and on the values of transfer coefficients.

For integration of transport equations, in the computational plane a grid (x_i,ω_j), was introduced, which divided the plane into $Nx \times Ny$ cells. The mesh size of the grid was chosen such that to provide for condensation of grid lines in flow regions with largest gradients. Values of the dependent variable Φ_{ij} were used as taken at grid nodes.

To integrate the equations in the transverse direction, we used a central compact-difference scheme of fourth-order accuracy [13], which, being put simply, on the analytical grid has the form

$$\begin{aligned} &\Phi'_{j-1} + 4\Phi'_j + \Phi'_{j+1} = \frac{3}{\Delta\omega}(\Phi_{j+1} - \Phi_{j-1}) + O(\Delta\omega^4) \\ &\Phi''_{j-1} + 10\Phi''_j + \Phi''_{j+1} = \frac{12}{\Delta\omega^2}(\Phi_{j+1} - 2\Phi_j + \Phi_{j-1}) + O(\Delta\omega^4) \end{aligned} \tag{3}$$

In the longitudinal direction the equations were integrated by the Crank–Nicholson method. Thus, the error in approximating the mesh values of the dependant variable, and the values of its first and second derivatives, was a value of order $O(\Delta x^2, \Delta\omega^4)$. The source member in the chemical kinetics was integrated using an implicit scheme of first-order accuracy. A comparison of obtained numerical data with available exact solutions for laminar boundary layer proved the method to be sufficiently accurate and reliable at an acceptable computing-cost level. From preliminary computations and from mesh refinement tests, the following values for the total number of nodes in x- and y-directions were chosen: $Nx = 1000$, $Ny = 150$.

4. Numerical data obtained for the boundary layer with combustion at different boundary conditions

The temperature profile is the most important parameter that characterizes the structure of near-wall flames. The calculated distributions of temperature over the thickness of the boundary layer are shown in Fig. 2 for different wall hydrogen concentrations (or for different blowing ratios $b_1 = 2j_W/\rho_0 u_0 c_f$, proportional to these concentration) at the boundary conditions of two types, $T_W = const$ (solid lines) and $T' = const$ (dotted lines). An increase in blowing intensity results in that the flame front gets forced from the wall both at constant wall temperatures and at constant injected-gas temperature. Simultaneously, in the case of $T' = const$ the wall temperature varies profoundly with blowing intensity, approaching the injected-gas temperature T' at heavy blowing. Here, although the flame-front temperature increases to finally approach the values typical of adiabatic combustion of hydrogen with air, the injected fuel acts as a coolant.

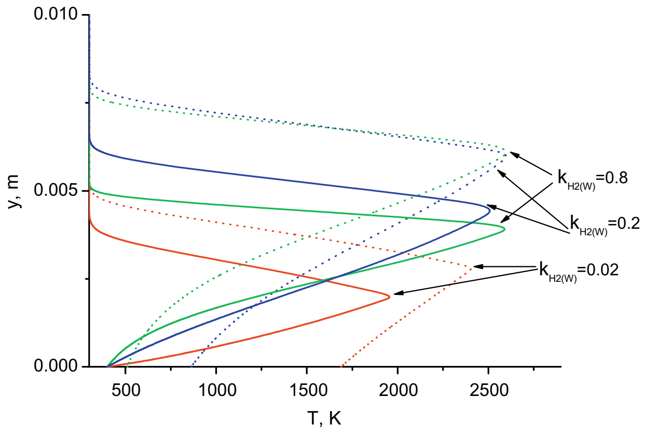

Fig. 2. Temperature profiles in the boundary layer. $Re_x = 10^5$. Solid lines: $T_W = const$; dash-and-dot lines: $T' = const$.

At constant injected-gas temperature $T' = const$ the flame-front coordinate is more remote from the wall in comparison with $T_W = const$. This also affects the distribution of dynamic characteristics of the boundary layer (see Fig. 3). Here, the boundary-layer thickness increases compared to the boundary layer without combustion, and the gradients of longitudinal flow velocity decrease in value, especially at constant injected-gas temperature.

These strong differences in the temperature and velocity profiles for the considered boundary conditions should also affect the behavior of surface friction and heat- and mass-transfer coefficients. Indeed, according to Fig. 4, the combustion in boundary layer leads to a substantial reduction of the friction coefficient $c_f/a = (\mu\partial u/\partial y)_W/\partial y)_W/\rho_0 u_0^2$.

The effect of friction reduction owing to intense heat release was analyzed in detail in [2] for turbulent flow. From Fig. 4,

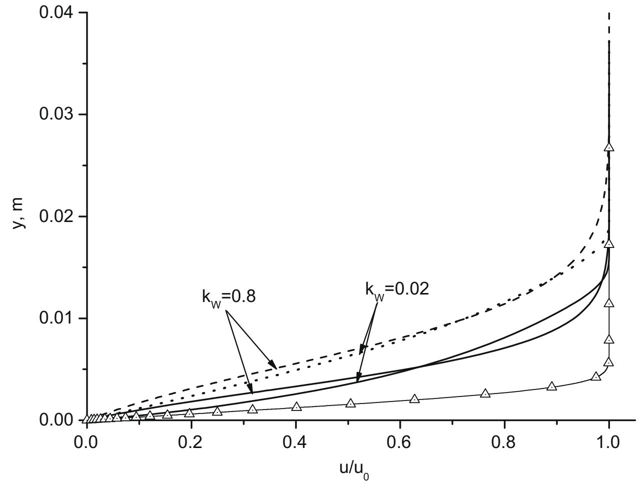

Fig. 3. Velocity profiles in the laminar boundary layer with combustion. Solid lines: $T_W = const$; dash line: $T' = const$; dot line: boundary layer without combustion yet with hydrogen blowing at $k_W = 0.02$.

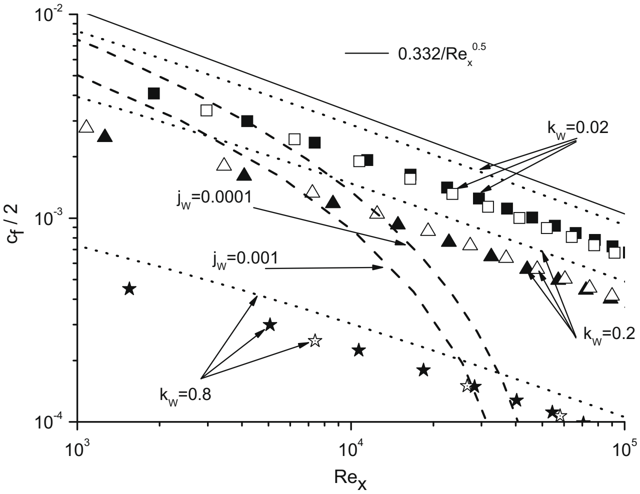

Fig. 4. Variation of the friction coefficient over the plate length. Lines: flow without combustion; full symbols: combustion with $T_W = const$; hollow symbols: combustion with $T' = const$.

another important conclusion can be drawn. Despite the considerable differences in the structure of boundary layers at $T' = const$ and $T_W = const$, there is no any appreciable difference between the friction coefficients. One possible reason for this unexpected behavior displayed by the friction coefficient is partial compensation of the effect of reduced flow velocity gradient on the wall shear stress and the gas viscosity increase resulting from higher level of wall temperature at gas blowing under $T' = const$.

In the case of $j_W = const$ the friction coefficient is not proportional to $\mathrm{Re}_x^{-0.5}$; it decreases over the plate length up to the point at which the boundary layer becomes blown off the wall. At this point, the friction coefficient is zero, $c_f/2 = 0$, and the wall fuel concentration is $k_W = 1$.

The above behavior, displayed by the shear stress, can be more clearly traced in Fig. 5, which shows the curves of the function of relative friction c_f/c_{f0} versus the blowing ratio $\bar{j}_W \mathrm{Re}_x^{0.5}$. Here, $c_f/2$ is the friction coefficient in the boundary layer with blowing and combustion, and $(c_f/2)_0 = 0.332\mathrm{Re}_x^{-0.5}$ is the friction on impermeable plate. The solid curve in Fig. 6 is the dependence for the function of relative friction [15]

$$c_f/c_{f0} = (1 - b/b_{cr})^{1,33} \tag{4}$$

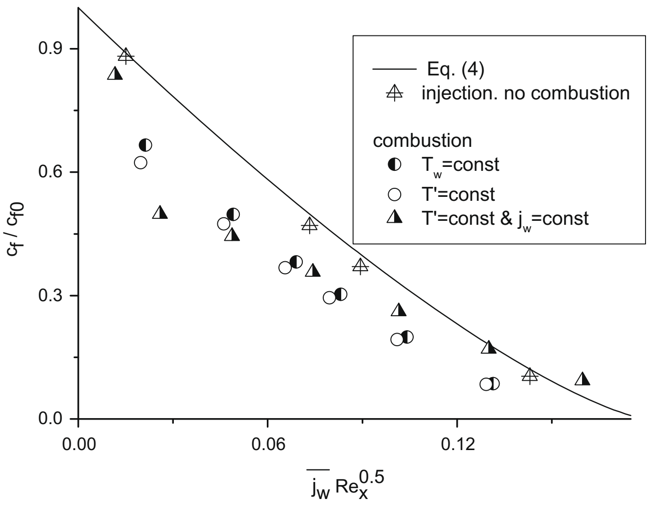

Fig. 5. Function of relative friction versus blowing ratio.

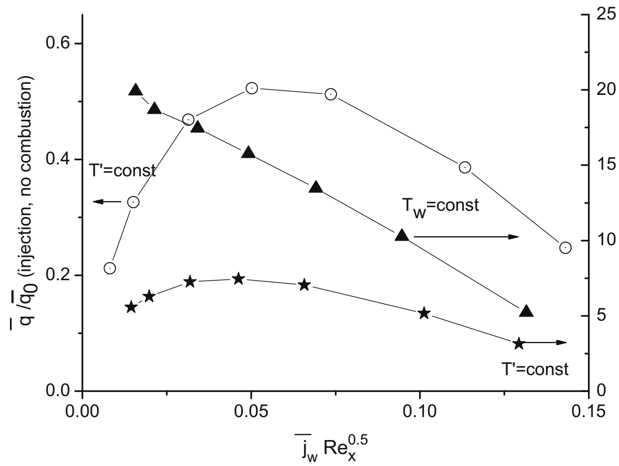

Fig. 6. Dimensionless wall heat flux versus blowing ratio in flows with and without combustion.

Here, b_{cr} = 0.18 is the critical blowing ratio for hydrogen injection into air calculated from the data of [15]. It should be noted that, in these coordinates, the friction coefficient values calculated for $k_W = const$ and $\bar{j}_W = const$ lie close together.

Numerical data obtained for the case of hydrogen blowing without combustion well agree with relation (4). The data obtained with combustion for different boundary conditions, although admitting generalization, lie notably lower than this curve, this being indicative of combustion-induced friction reduction. Yet, the friction reduction in laminar flow is less profound in comparison with turbulent flow [2]; in the latter case the friction reduction factor can be 2 or greater. This again lends support to the conclusion that non-isothermal conditions affect laminar boundary layer lesser than turbulent boundary layer [14].

5. Effect of boundary conditions on the distribution of heat fluxes in laminar boundary layer with combustion

The distribution of heat fluxes and their dependence on blowing intensity and on thermal and concentration boundary conditions is of particular interest for designs of apparatuses with boundary-layer combustion. It was shown earlier (see [9]) that in the case of porous blowing the boundary conditions at the wall may have a dramatic influence on the laws of heat and mass transfer. For instance, at blowing with constant wall temperature the heat flux $q_W = -(\lambda \partial T/\partial T)_W$, like friction, decreases monotonically with blowing intensity throughout the whole range of blowing ratios until, finally, the regime with boundary layer blown off the wall sets in. At constant injected-gas temperature, in which case the wall temperature depends on blowing intensity, the heat flux first increases to reach its maximum value, and then decreases. This phenomenon can be attributed to the fact that at low blowing intensities the wall temperature is close to the main-flow temperature, this implying a low temperature gradient and, hence, a small heat flux; on the other hand, at high blowing intensities the boundary layer gets blown off the wall the wall, making the wall heat flux zero.

A similar behavior of the heat fluxes is also observed with blowing of reacting substances. The relative heat flux $\bar{q}/\bar{q}_0$ versus the blowing ratio $\bar{j}_W \mathrm{Re}_x^{0.5}$ at different boundary conditions is shown in Fig. 6. Here, $\bar{q} = q_W/\rho_0 u_0 c_{p0}(T_0 - T')$ is the dimensionless heat flux through porous wall, which resembles the Stanton number, and $\bar{q}_0 = 0.332\mathrm{Re}_x^{-0.5}\mathrm{Pr}^{2/3}$ is the dimensionless heat flux through impermeable plate.

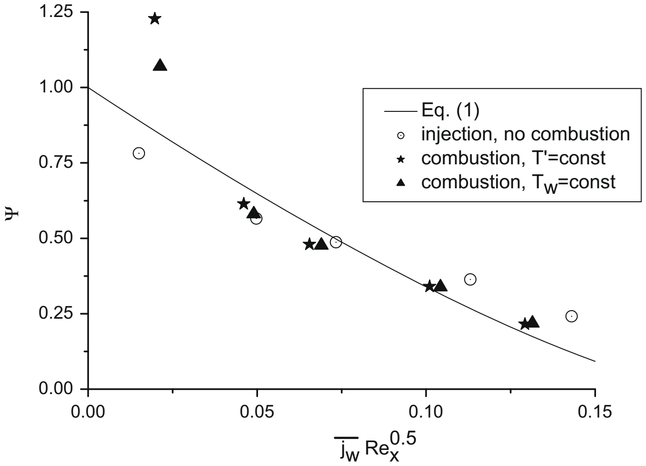

Fig. 7. Function of relative heat transfer versus blowing ratio.

As it could be expected, at gas blowing with constant injected-gas temperature both with and without combustion the curves exhibit a maximum reached, in the regimes with $j_W = const$ and $k_W = const$, approximately at the same values of the blowing ratio, the maximum heat flux in the case with combustion being one order of magnitude greater than its value in the case of non-reacting flow. The latter can be attributed to a zone with intense heat release (flame front), present in the boundary layer, that appreciably intensifies heat- and mass-transfer processes. At constant wall temperature the heat flux displays monotonic reduction whose value turns out to be slightly higher than that in the case of constant injected-gas temperature because of lower wall temperature. The function of relative heat transfer $\Psi_T = \mathrm{St}/\mathrm{St}_0$ versus blowing intensity is shown in Fig. 7. It should be noted that, here, it is the flame-front temperature that was chosen as the characteristic temperature, so that $\mathrm{St} = -(\lambda \partial T/\partial T)_W/\rho_{fr} u_{fr} c_{p0}(T_{fr} - T_w)$. As it is evident from the graph, the values obtained with and without combustion lie close together, this being indicative of the fact that heat-transfer processes proceeding inside the boundary layer weakly affect the heat transfer. This result is interesting from the viewpoint of engineering calculations.

6. Conclusions

Laminar boundary layer with porous blowing and combustion of a fuel was simulated numerically. Different types of thermal and diffusion boundary conditions were analyzed. Combustion is shown to result in a significant reduction of the friction coefficient and in an increase of wall heat fluxes. However, the reduction of friction resistance in laminar boundary layer turned out to be less pronounced than that in turbulent boundary layer because of weaker influence due to gas-mixture density gradient over the boundary-layer thickness in laminar flow. The friction coefficients in regimes with constant wall temperature and constant injected-gas temperature were found to have close values despite the fact that significant differences in velocity and temperature profiles were observed in these regimes. As in the case without the chemical reaction, for the combustion with constant injected-gas temperature the heat flux versus blowing intensity displays a maximum, the amplitude of this maximum being almost one order higher than in the case without combustion.

Acknowledgments

The work was supported by the Russian Foundation for Basic Research (Grant 06-08-01149-a), and by the Program for Supporting Scientific Schools in Russian Federation (Grant NSh-2855.2008.8).

References

[1] B.F. Boyarshinov, E.P. Volchkov, V.I. Terekhov, Heat and mass transfer in boundary layer with ethanol blowing and combustion, Combust. Expl. Shock Waves 30 (1) (1994) 7–14.
[2] B.F. Boyarshinov, E.P. Volchkov, V.I. Terekhov, Boundary-layer structure with hydrogen combustion with different blowing intensities, Combust. Expl. Shock Waves 38 (3) (2002) 269–277.
[3] D.B. Spalding, A standard formulation of the steady convective mass transfer problem, Int. J. Heat Mass Transfer 1 (1960) 192–207.
[4] D.B. Spalding, Convective Mass Transfer: An Introduction, Edward Arnold, London, 1963.
[5] T. Ueda, M. Mizomoto, S. Ikai, Velocity and temperature fluctuations in a flat plate boundary layer diffusion flame, Combust. Sci. Technol. 27 (3–4) (1982) 133–142.
[6] C. Yam, H. Dwayer, An investigation of the influence of blowing and combustion on turbulent boundary layer, AIAA Paper 226, 1987, pp. 1–8.
[7] T. Hirano, Y. Kanno, Aerodynamic and thermal structures of the laminar boundary layer over a flat plate with a diffusion flame, in: Proc. 14th Intern. Symp. on Combustion, 1973, pp. 391–398.
[8] T. Ueda, A. Ooshima, N. Saito, M. Mizomoto, Aerodynamic structure of a laminar boundary layer diffusion flame over a horizontal flat plate (Experimental analysis), JSME Int. J. Ser. II 34 (4) (1991) 527–532.
[9] E.P. Volchkov, Concerning the heat and mass transfer features on permeable surfaces, Int. J. Heat Mass Transfer 49 (2006) 755–762.
[10] R.C. Reid, J.M. Prausnitz, T.K. Sherwood, The Properties of Gases and Liquids, McGraw-Hill, New York, 1997.
[11] W.C. Rivard, O.A. Farmer, T.D. Butler, RICE: A computer program for multi-component chemically reactive flows at all speeds, Los Alamos Scientific Laboratory Report, LA-7427, 1979.
[12] S.V. Patankar, D.B. Spalding, Heat and Mass Transfer in Boundary Layers, Intertext, London, 1970.
[13] S.K. Lele, Compact finite-difference schemes with spectral-like resolution, J. Comput. Phys. 103 (16) (1992) 16–42.
[14] S.S. Kutateladze, Fundamentals of Heat Transfer, Edward Arnold, London, 1963.
[15] V.M. Eroshenko, L.I. Zaichik, Hydrodynamics and Heat and Mass Transfer on Permeable Surfaces, Nauka, Moscow, 1984 (in Russian).